WBPH036-05 Tutorial 6 30-03-2022

Astroparticle Physics

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There are five exercises, and you can solve up to two exercises. You have 45 minutes to solve every exercise. If solving only one exercise, you should complete the assignment by 15:45, and if solving two exercises, you should hand in the assessment by 16:30. The grade in each exercise will replace your original grade (if any) only if the new grade is higher than the previous one. Offline resources (the textbook, the reader, the lecture slides and your notes) are allowed, and online resources such as Google are not allowed. These exercises can only be solved on campus, and documents uploaded on Dropbox will not be graded. Please write your name and S-number on every sheet.

The final grade for each exercise is calculated as: $\sum \text{points} + 1$. There are 9 total points per each exercise.

Question 1: Interactions of particles and radiation
Consider the interaction of ultra high-energy cosmic-ray protons with cosmic microwave background (CMB) photons.

(a) (1 pt)
Write explicitly the initial state and at least one of the possible final states.

(b) (2 pts)
Calculate the four momentum in the lab frame for the initial state.

(c) (2 pts)
Calculate the four momentum in the COM frame for the final state. *Hint: the particles still have energy in the COM frame.*

(d) (2 pts)
Derive an expression for the energy threshold for the incoming proton to undergo this process, in terms of the CMB photon energy E_γ, the pion mass m_π, and the proton mass m_p.

(e) (2 pts)
Derive an expression for the energy threshold of a cosmic-ray iron (Z=26, A=56), and compare it with the previous result. Using the two expressions, and the values given below, give an order of magnitude for the energy threshold of the UHECR proton, and the cosmic-ray iron (assume a head on collision with the CMB photon).

Useful values and approximations:

- $m_p = 0.938\ \text{GeV}/c^2$ (assume $m_p \sim m_n$);
- $m_{\pi^+} = 0.140\ \text{GeV}/c^2$ and $m_{\pi^0} = 0.135\ \text{GeV}/c^2$;
- $E_{\gamma_\text{CMB}} = 6.34 \times 10^{-13}\ \text{GeV}$

- $v_p \sim c$.

Question 2: interactions of high-energy muons

The detection of high-energy muons is a very powerful tools in experimental astroparticle physics.

(a) (4.5 pts)
High-energy muons traversing matter lose energy according to

$$-\frac{1}{\rho}\frac{\mathrm{d}E}{\mathrm{d}x} \approx a + bE,$$

where a describes the ionisation energy loss and b describes the loss due to bremsstrahlung and e^+e^- pair-production processes. For rock, taken to have $\rho = 2.65\,\mathrm{g\,cm^{-3}}$, the parameters a and b have values $a \approx 2.5\,\mathrm{MeV\,cm^2\,g^{-1}}$ and $b \approx 3.5 \times 10^{-6}\,\mathrm{cm^2\,g^{-1}}$ and can be considered energy independent.

i) (1.5 pts) Compute the energy at which the ionisation and bremsstrahlung/pair-production processes are equally important.

ii) (3.0 pts) Compute the muon range in rock with, based on the energy loss formula given above. Assume a muon energy of $E_\mu = 100\,\mathrm{GeV}$.

(b) (2.5 pts)
The number of Cherenkov photons N produced per unit length x can be approximated as

$$\frac{\mathrm{d}N}{\mathrm{d}x} \approx 490 \cdot z^2 \sin(\theta_C) \cdot \mathrm{cm}^{-1}$$

with the charge z of the incoming particle, the Cherenkov angle $\theta_C = \arccos(\frac{1}{n\beta})$, the refraction index of the material n and $\beta = v/c$. Consider a muon with total energy $E_\mu = 1\,\mathrm{GeV}$ that travels through a water Cherenkov tank with a thickness of $\Delta x = 5\,\mathrm{m}$. How many Cherenkov photons are induced by the muon?
Hint: The refraction index of water is given by $n = 1.3$ and the muon mass by $m_\mu = 105\,\mathrm{MeV}$.

(c) (2.0 pts)
The IceCube experiment is a neutrino observatory with Cherenkov detectors being located under the Antarctic ice. Explain how IceCube can measure neutrinos via Cherenkov emission despite the fact that neutrinos do not have an electric charge.

Question 3: ground-based gamma-ray detectors

Consider a photon entering the atmosphere vertically with initial energy $E_0 = 1\,\mathrm{TeV}$. For electromagnetic showers in air $E_c = 100$ MeV. (critical energy)

(a) (2 pts)
Compute the number of particles at the shower maximum.

(b) (1 pt)
How is the critical energy in a particle shower defined?

(c) (2 pts)
For a vertical shower, the depth x in the upper atmosphere (in g/cm^2) can be expressed as a function of height h using the following expression:

$$X = Ce^{-h/H}$$

where $C \simeq 1030$ g/cm^2, $H = 7$ km; consider $X_0(air) = 36.6$ g/cm^2.
Calculate the maximum depth height h_{max} in the Heitler model.

(d) (2 pts)
Define the concept of the *light pool*, and compute the area of the light pool assuming the Cherenkov light to be emitted at height h_{max} with an aperture angle of 1.3 degrees.

(e) (2 pts)
In short, describe *one* currently operating ground-based experiment which is able to measure TeV-range gamma rays.

Question 4: Ground-based cosmic-ray detectors
Cosmic rays of the highest energies can only be detected with ground-based experiments.

(a) (2 pts)
Describe the phenomenon of Extensive Air Showers (EAS) in the atmosphere, and list which particles can be detected on the ground.

(b) (1 pts)
Discuss how it is possible to discriminate between the electromagnetic and the hadronic showers, based on the particles detected on the ground.

(c) (2 pts)
Describe the Pierre Auger experiment and its main scientific goals.

(d) (2 pts)
The figure 1 shows $< X_{max} >$ and its fluctuations as a function of energy. Discuss the plot and its implications.

(e) (2 pts)
Explain what limits the energy measurement (on the horizontal axis) both a the lower and upper ends.

Question 5: cosmic-ray sources, propagation and acceleration
Several strong arguments suggest that the fast shock wave in the remnants of exploding stars, such as Supernova Remnants (SNRs) could be one of the acceleration sites of cosmic rays in our galaxy.

(a) (1.5 pts)
Briefly explain how SNRs could accelerate cosmic rays up to 10^{15} eV.

(b) (1 pt)
Explain what is the main role of magnetic fields in the acceleration process.

(c) (1.5 pts)
Shortly discuss the 1st order Fermi mechanism for cosmic-ray acceleration.

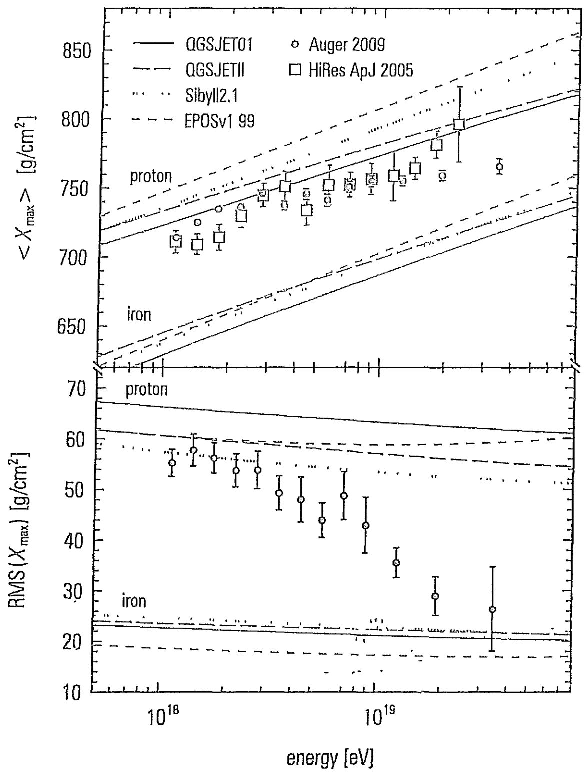

Figure 1: $< X_{max} >$ and its fluctuations as a function of energy. Figure taken from Grupen, Astroparticle physics, Springer 2020

(d) (2 pts)
Compute the number of accelerations cycles that a particle with initial energy $E_0 = 0.1\ MeV$ must undergo in a shock wave of a SNR (1st order Fermi mechanisms) to reach $E = 10\ TeV$. Assume $\beta = 10^{-4}$ for the velocity of the astrophysical object (shock wave).

(e) (1 pts)
Discuss the conventional model that describes the cosmic rays up to the knee.

(f) (2 pts)
Fig. 2 represents the cosmic-ray flux for energies between 10^{16} and 10^{18} eV measured by the KASCADE-Grande experiment. Discuss the plot and relate it to some of the features in the spectrum and to the chemical composition of cosmic rays.

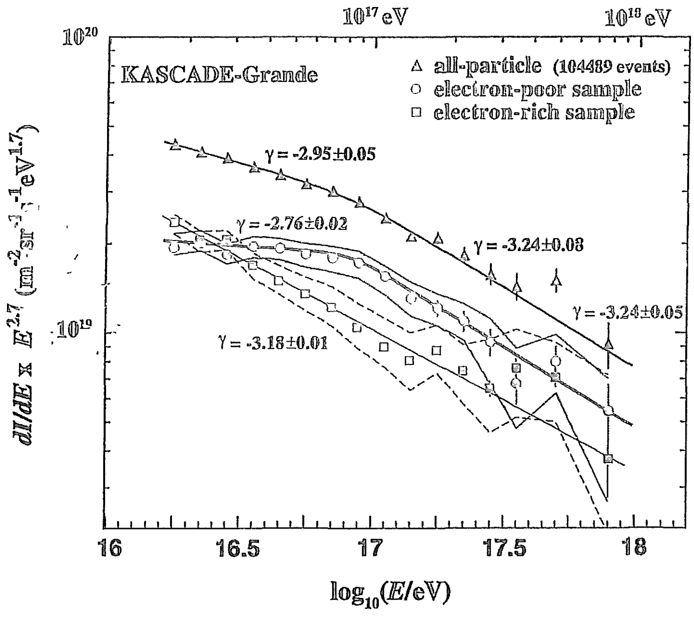

Figure 2: KASCADE-Grande measurements of the cosmic-ray flux. Figure taken from Apel et al (KASCADE Collaboration) Physical Review Letters 107 (2011) 171104